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# Critical Clearing Time Estimation for Power Systems Based on Wide Area Measurements

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**ABSTRACT** This paper introduces a new algorithm based on the properties of the solution of the eigenvalues and the decomposition of the singular values to establish a limit that allows to determine the stability of the system and calculate the critical clearing time of fault from measurements of the variables of interest. The dynamics of the electrical power system is represented by the behavior of the dominant eigenvalue and the critical clearing time is calculated by comparing this dynamic with a threshold derived from the maximum singular value. This algorithm does not require knowing any parameter of the power system and does not need to solve the algebraic differential equations that describe the dynamics of the synchronous generators. Moreover, it provides an analytical expression for the threshold that allows the calculation of the critical clearing time from a set of measurements. This methodology is tested in a single machine infinite bus system and shows results with acceptable precision and has the possibility of being implemented in a real-time scheme.

**KEYWORDS** Critical clearing time, eigenvalue analysis, power system stability, phasor measurement units, singular value decomposition.

## I. INTRODUCTION

Transient stability is concerned with the ability of the power system to maintain synchronism when subjected to a severe disturbance, such as a short circuit on a transmission line [1]. The critical clearing time (CCT) of a fault can be defined as the maximum allowable value of the clearing time for which the system remains stable after a disturbance and it is one of the most relevant metrics in transient stability assessment as an index of stability [2]. The estimation of an accurate value of the CCT of a fault allows operators to take control actions and avoid instability conditions.

The methodologies applied to estimating the CCT of fault can be classified into two main groups. In first group, time domain simulation, the power system is represented by a set of algebraic differential equations which are solved using numerical methods. Although this method allows accurately computing the value of the CCT, it consumes a large amount of time in the process [3], [4]. In the second group, the direct method, the swing equations are translated into an energy reference frame to obtain the critical energy limit of the whole system during a disturbance [5], [6].

For both methods, it is necessary to solve a certain number of equations and sometimes, it results difficult because of the number of equations increases proportionally to the size of the network. Furthermore, the analysis is mostly focused to a power system representation in which the generator has represented by the classical model and loads are modelled as constant impedances.

Some disadvantages, such as the unknown parameters of the power system, the high computational burden, quite conservative results by using an equivalent model of the power system, growing the need of developing new methodologies to estimate the critical clearing time of a fault.

Recently, several methods have adopted the eigenvalue analysis and the singular value decomposition (SVD) method to analyze the information of a high dimensional dataset and project it onto a lower dimensional space, with a minimum loss of information. In the transient stability assessment, the singular value decomposition method has been used to identify coherent groups [7]. Others closely related methods to it has been used also; principal component analysis is applied to detect and extracting anomalous dynamic events from measured data [8], [9] and proper orthogonal decomposition [10] is used to obtain information from dynamic patterns and information about each one of the dominant modes. In voltage stability assessment, SVD has been used as a voltage security index by examining the smallest singular value of the Jacobian [11], moreover, analyze the information of the inverse Jacobian matrix and identify the most sensitive buses base on the largest singular values [12].

Accordingly, in this paper, a new algorithm to estimate the CCTs of a power system is introduced. The measurement data will be analyzed through the dominant eigenvalue. The variation of the eigenvalues with respect to the changes in the system will be bounded by a threshold, obtained from the calculation of the singular value decomposition method analyzed, and it will be used to estimate the CCT of a fault, considering if the system is under stable operating conditions or if the system loses stability when subjected to a given disturbance. The system model is constructed with measurements of certain power system variables by using wide area measurement system.

This paper is organized as follows. In Section II, the variables measured from wide area measurement system and the procedure used to form the measurement matrix from a sliding window are described. In this section, the singular value decomposition and a discussion of the characteristics of the maximum singular values and its interpretation are also described. The proposed algorithm is presented in Section III, followed by its application to a test system with simulation results presented in Section IV. Finally, the conclusion is presented in Section V.

## II. ANALYTICAL FORMULATION

### A. ENSEMBLE MATRIX

To obtain reliable results in transient stability analysis is necessary to have an accurate model of the power system that contains the information of the network connection and the parameters of the elements that make it up. However, having a realistic model of the power system represents a great challenge due to the constant change of the network connection, the dynamics of the loads and the complex models of the transmission devices like SVCs and other FACTS. Due to the difficulty of measuring angle and frequency of the synchronous generator rotor in a real system, measurements of voltage phasor angle dynamics at the generation buses are used to form the ensemble matrix, which represents the dynamic of the all power system.

The ensemble matrix, $\boldsymbol{X} \in \mathbb{R}^{M\times n}$ is formed by vectors, $\boldsymbol{V}_i = [v(x_i,t_1) \quad v(x_i,t_2) \quad \cdots \quad v(x_i,t_M)]^T$ , $i = 1,\dots,n$ , using a sliding window and representing a set of snapshots from the $i-th$ measured variable at the $M-th$ time. The ensemble matrix can be written as,

$$\boldsymbol{X} = [\boldsymbol{v}_1 \quad \boldsymbol{v}_2 \quad \cdots \quad \boldsymbol{v}_n] = \begin{bmatrix} v(x_1,t_1) & \cdots & v(x_n,t_1) \\ \vdots & \vdots & \vdots \\ v(x_1,t_M) & \cdots & v(x_n,t_M) \end{bmatrix} \quad (1)$$

where $M$ is the number of samples of the sliding window an $n$ is the number of measured variables.

### B. SINGULAR VALUE DECOMPOSITION

For a real measurement matrix, $\mathbf{X} \in \mathbb{R}^{M\times n}$, with rank $r$ ($m \geq n$ and $r \leq n$), there exist orthogonal matrices $\boldsymbol{U} \in \mathbb{R}^{m\times m}$ and $\boldsymbol{V} \in \mathbb{R}^{n\times n}$ such that,

$$\boldsymbol{X} = \boldsymbol{USV}^T \quad (2)$$

where $\mathbf{V}^T$ is the conjugate transpose of $\mathbf{V}$ and $\mathbf{S}$ is a pseudo-diagonal and semi-positive definite matrix. The columns of $\mathbf{U}$ are called the left singular vectors, the rows containing the elements of $\mathbf{V}^T$ are the right singular vectors, and $\mathbf{S}$ contains the singular values denoted as $\sigma_1, \sigma_2, \dots, \sigma_r$. Furthermore, $\sigma_k > 0$ for $1 \geq k \geq r$ and $\sigma_k = 0$ for $(r+1) \geq k \geq n$ .

Moreover,

$$\boldsymbol{X}^T\boldsymbol{X} = (\boldsymbol{USV}^T)^T(\boldsymbol{USV}^T) = \boldsymbol{VS}^T\boldsymbol{SV}^T \quad (3)$$

in which $\mathbf{S}^T$ denotes the transpose of $\mathbf{S}$. According to (3), the left and right singular eigenvectors of $\mathbf{X}$ are the eigenvectors of $\mathbf{XX}^T$ and $\mathbf{X}^T\mathbf{X}$, respectively. Moreover, the singular values of $\mathbf{X}$ are found to be the square roots of the eigenvalues of $\mathbf{XX}^T$ or $\mathbf{X}^T\mathbf{X}$.

The symmetric matrix $\mathbf{X}^T\mathbf{X}$, has a complete set of orthonormal eigenvectors $\mathbf{x}_i$, i.e. $\mathbf{x}_i^T\mathbf{x}_i$ =1 and $\mathbf{x}_i^T\mathbf{x}_j$ =0 for $i \neq j$ , which are into the columns of $\mathbf{V}$,

$$\boldsymbol{X}^T\boldsymbol{X}\boldsymbol{x}_i = \lambda_i\boldsymbol{x}_i \quad (4)$$

where $\lambda_i$is the $i-th$ eigenvalue of $\mathbf{X}^T\mathbf{X}$. Taking the inner product with $\mathbf{x}_i$ and applying the properties of the Euclidean norm the following expression is obtained:

$$\|\boldsymbol{X}\boldsymbol{x}_i\|^2 = \lambda_i \quad (5)$$

For each nonzero eigenvalue of the matrix $\mathbf{X}^T\mathbf{X}$ there is a singular value ($\sigma_i$) represented by

$$\sigma_i = \sqrt{\lambda_i} \quad (6)$$

Generally, singular values reveal how much stretch or compress can present an eigenvector under a transformation by an arbitrary matrix, i.e., the matrix $\mathbf{X}$ maps a unit sphere in $m$ dimension space to an ellipsoid in $r$ dimension space with the directions indicated by left singular vectors and magnitudes of singular values.

From (7), it can be observed that obtaining the maximum singular value, $\sigma_{max}$, of a matrix allows determining a limit of the magnitude that can take the eigenvalues of this matrix and it can be expressed as follows,

$$\lambda_i \leq \sigma_{max} \quad (7)$$

On this basis, in this paper, the information of the measurement matrix, $\mathbf{X}$, is extracted by computing the SVD to obtain the maximum singular value and the dominant eigenvalue.


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## III. ESTIMATION OF CCT BASE ON SVD

### A. COMPUTE THE THRESHOLD

From (7), the maximum limit that the magnitude of the eigenvalues of a measurement matrix can take for the not faulted system is used as a reference to identify the steady-state operation condition and to compute the CCT. The procedure to compute the threshold can be summarized as follows:

1. From a given measurement matrix, $\boldsymbol{X}$, containing the measured variables of a set of reference signals, compute the singular value decomposition as in (2).
2. Obtain the maximum singular value.
3. Repeat steps 1 and 2 along the entire reference signal.
4. Compute the threshold as follows,

$$\text{Threshold} = \frac{1}{k}\sum_{i=1}^{k}\sigma_{\max i} \quad (8)$$

where $k$ represents the number of sliding windows required to process the reference signal.

### B. ESTIMATION OF CRITICAL CLEARING TIME

Due the threshold is calculated from a set of reference signals measured in steady-state operation (see (8)), the magnitude of the dominant eigenvalue $(\lambda)$ must be smaller than the magnitude of the threshold when the power system is in steady-state operation. It has variations, about $\nu = 1\times10^{-4}$ (heuristic value obtained after multiple simulations), due to the dynamic of the loads and the control of the synchronous generators. When the power system is disturbed, the magnitude of $\lambda$ increases and may exceed the limit established by the threshold (8). Therefore, to identify if a disturbance occurs in the system is necessary to compute the ratio of change of $\lambda$ as follows,

$$\Delta\lambda = \frac{d\lambda}{dt} \quad (9)$$

when $\Delta\lambda$ reaches a value greater than $\nu$ a disturbance is detected in the power system (see Fig. 1).

The critical clearing time is estimated by calculating the time required for $\lambda$ to cross the threshold from the onset of the disturbance,

$$t_{CCT} = t_c - t_s \quad (10)$$

where $t_s$ is the fault starting time and $t_c$ is the crossing time.

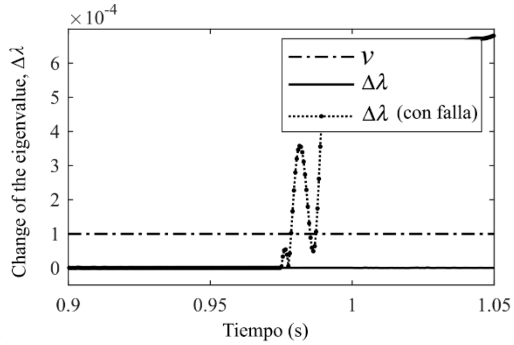


**Fig. 1.** **Ration of change of the dominat eigenvalue, $\Delta\lambda$, for a steady-stable condition and a fault condition.**

## IV. RESULTS

To evaluate the performance of the proposed algorithm for estimating the critical clearing time, a single machine infinite bus test system is used. A sampling frequency of 2.048 kHz was used along with a 256-sample sliding window and it is considered a three-phase fault as the disturbance. The critical clearing time is calculated using the proposed algorithm and then is compared to that obtained from the time domain simulation method (TDSM) and the relative error $(\varepsilon)$ is obtained by the following equation:

$$\varepsilon = \left|\frac{t_{\text{CCT}} - t_{\text{TDSM}}}{t_{\text{TDSM}}}\right| \times 100\% \quad (11)$$

where $t_{CCT}$ is the time calculated by the proposed algorithm and $t_{TDSM}$ is the time obtained by the TDSM.

The test system is shown in Fig. 2. The generator is connected to a 400kV network through a 175MVA, 400/14.7kV, 50Hz, transformer with a Δ−Y connection and two transmission lines represented by the Π model. Generator parameters: $H$ =0.02151 s, $R_a$=0.0015 p.u., $X_0 = X_l$ = 0.1 p.u., $X_d$ = 1.13 p.u., $X'_d$ = 0.3 p.u., $X''_d = X'_q$ = 0.2 p.u., $T'_{d0}$ = 6 s, $T''_{d0}$ = 0.05 s, $X_q$ = 0. 66 p.u., $T'_{q0}$ = 0.12 s. Transformer parameters: $R$ = 0.003 p.u., $X$ = 0.12 p.u.. Transmission lines parameters: $R_0$ =1.036 Ω, $L_0$ = 751.209 mH, $C_0$ = 2.05 mF, $R_1$ = 3.752, $L_1$ = 197.565 mH, $C_1$ = 3.30 mF.

The estimated threshold, using (8), for the test system is Threshold = 0.4076. A three-phase fault is put on Bus 2 and the transmission line that is put out of service is Line 2. Two scenarios are evaluated:

*1) Scenario 1:* The generator control system is modeled as a simple excitation system (SEXS). In this scenario the behavior of λ is evaluated when the system is stable and unstable, and the times used for the calculation of the CCT are shown in Fig. 3. When the system is stable, the magnitude of $\lambda$ remains below the threshold, whereas when the system is unstable the magnitude of $\lambda$ becomes larger and exceeds the value of the threshold. For this scenario the estimated critical clearing times is $t_{CCT} = 0.260\ s$.


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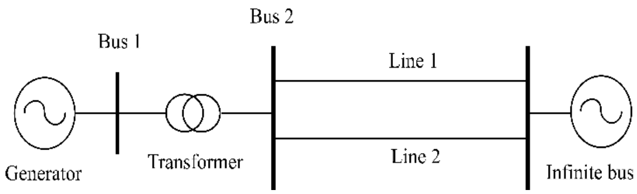



Fig. 2. **Single machine infinite bus test system.**

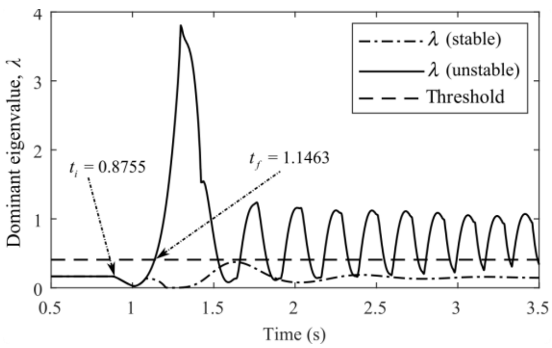



Fig. 3. **Behavior of the dominant eigenvalue, $\lambda$, and the identification of the fault starting time ($t_s$) and the crossing time ($t_c$) for Scenario 1.**

*2) Scenario 2:* The control system includes an automatic voltage regulator (AVR). The behavior of $\lambda$ and the identification of the fault starting time ($t_s$) and the crossing time ($t_c$) are presented in Fig. 4. The estimated critical clearing times is $t_{CCT} = 0.271\ s$.

Fig. 3 and Fig. 4 show that the magnitude of $\lambda$ in pre-fault state is practically constant. Once the fault starts, the magnitude of $\lambda$ begins to change. When the system is stable, in both scenarios, the magnitude of $\lambda$ does not exceed the threshold value. On the other hand, when the system is unstable, for Scenario 1 the magnitude of $\lambda$ reaches a maximum value and then oscillates exponentially increasing. However, in Scenario 2 the transient response of $\lambda$ has damped oscillations once it reaches its maximum value. In both scenarios, the magnitude of $\lambda$ exceeds the threshold value. Having said that, it is demonstrated that the behavior of $\lambda$ represents the dynamics of the angles of the generators of the power system, and reflects the changes in the parameters and the topology of the system.

To verify the results obtained is carried out the phase plane for the single machine infinite bus test system, *Scenario 1* for both operation conditions: stable and unstable. In normal conditions, the power system operates at the pre-fault equilibrium point (square mark). As the disturbance starts, the operating point has an excursion in the phase plane and moves away from the pre-fault equilibrium point. After the fault is cleared, the power system reaches a new equilibrium point (triangular mark) due to topological changes in the system. For the stable case (see Fig. 5(a)) the new stable point is very close to the original one indicating that the post-fault equilibrium point is stable. However, for the unstable case (see Fig. 5(b)) the new stable point is too far from the original one, i.e., the post-fault equilibrium point left the region of attraction of the original stable equilibrium point. This test shows that the algorithm correctly estimates the stability of the power system.

It is demonstrated that the algorithm correctly estimates if the system is stable or unstable when it is subject to a disturbance. The results obtained from the calculation of the fault CCT are concentrated in Table I. Here, it is observed that the times obtained with the proposed algorithm are smaller than the times obtained by TDS method. The average error for this test system is 7.436%. With the obtained results, it can be concluded that the algorithm works correctly in this test system for calculating the CCT of fault for the presented scenarios and discriminates correctly if the system is stable or unstable.

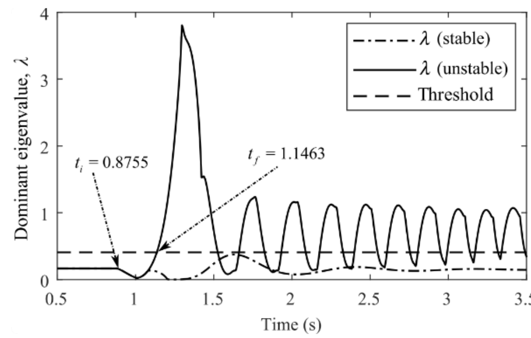
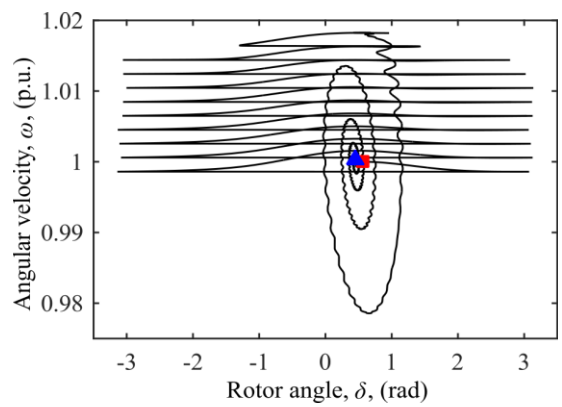
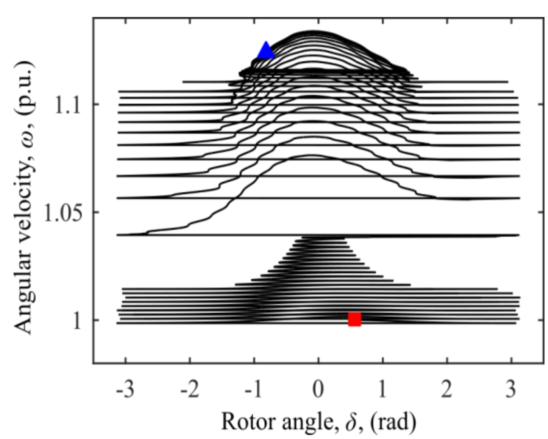



Fig. 4. **Behavior of the dominant eigenvalue, $\lambda$, and the identification of the fault starting time ($t_s$) and the crossing time ($t_c$) for Scenario 2.**




(a)




(b)

Fig. 5. **Phase plane of the single machine infinite bus test system for the Scenario 1: (a) stable, (b) unstable.**

## V. CONCLUSIONS

The proposed algorithm for the calculation of the critical clearing time of a fault based on the properties of the solution of the eigenvalues and the decomposition of the singular values estimates correctly the CCT for different cases of study. The calculated relative error does not exceed 10%. Therefore, it is concluded that accuracy of the proposed method is acceptable. This algorithm compared with the TDMS shows outstanding advantages. It captures the entire dynamics of the power system, unlike other methods in which several simplifying assumptions are made. In addition, does not require knowing the topology or the parameters of the power system. The formulated threshold depends only on the measurements obtained from a stable operating condition of the power system and its analytical expression allows it to be adaptable.

**TABLE I. THE RELATIVE ERROR OBTAINED BY COMPARING THE CCT ESTIMATED BY THE PROPOSED ALGORITHM AGAINST THE OBTAINED BY TDSM.**

| Case | CCT (s) | | $\varepsilon$ (%) |
|---|---|---|---|
| | *TDSM* | *Proposed algorithm* | |
| 1 | 0.282 | 0.260 | 7.626 |
| 2 | 0.292 | 0.271 | 7.246 |

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